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Strings at the bottom of the deformed conifold

Jose J. Blanco-Pillado and Alberto Iglesias

Center for Cosmology and Particle Physics
Department of Physics, New York University
New York, NY, 10003, U.S.A.
E-mail: blanco-pillado@physics.nyu.edu, iglesias@physics.nyu.edu

Abstract: We present solutions of the equations of motion of macroscopic F and D strings extending along the non compact 4D sections of the conifold geometry and winding around the internal directions. The effect of the Goldstone modes associated with the position of the strings on the internal manifold can be seen as a current on the string that prevents it from collapsing and allows the possibility of static 4D loops. Its relevance in recent models of brane inflation is discussed.

Keywords: Superstrings and Heterotic Strings, D-branes, Cosmology of Theories beyond the SM.

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Contents

1. Introduction

There has been a considerable amount of effort recently in trying to embed inflation within the context of string theory. All these models are based on the simple realization that moduli associated with the ingredients necessary to go from ten to four dimensions will be seen as light fields in the four dimensional description. In this regard, we can distinguish between two different origins for the would be inflaton.

On the one hand, compactifications mechanisms generate many moduli that come from different components of the metric, the dilaton, or the different antisymmetric forms present in the low energy description of string theory in ten dimensions. But these moduli should be fixed today by their respective potentials, so one of the most simple frameworks to get inflation would be to use these potentials to create a period of inflationary expansion. This idea has been pursued within string theory for a long time [1] and has its most recent incarnation in [2].

On the other hand, the discovery of D-branes has opened up a new opportunity to get inflation within string theory [3 − 5]. The idea is that we can use the moduli that parametrizes the distance between the branes along the compact directions as the inflaton field. In [6] these type of models were discussed within the framework of compactification with fluxes [7]. The authors argue that due to the potentials for the D-branes coming from the background geometry, it would be natural to have this process of inflation happening when the branes are located on a warped region of the internal manifold. These regions, called throats can be thought as regularized deformations of the conifold singularity [8] on a Calabi-Yau. In [9] the authors were able to find an exact solution of the supergravity equations of motion that describe the geometry once some fluxes are turned on along the cycles of the internal space. We will review this solution in section III of this paper since it is important for our purposes.

Brane inflation ends in these scenarios when the branes collide at the bottom of this warped geometry. It has been suggested that brane annihilation would leave behind a network of lower dimensional extended objects [10 − 13], which would be seen as strings from the four dimensional point of view. This alternative has renewed the interest on the possibility of observing cosmological consequences of cosmic strings, either fundamental strings (F strings) or D1-branes (D strings).

On the other hand, many high energy extensions of the standard model predict that our universe underwent a phase transition during which one dimensional topological defects of the field theory in question could have been produced [14, 15]. These would be the usual cosmic strings that have been extensively studied in the past in relation with different cosmological and astrophysical observations [16, 17]. It is therefore interesting to look for distinguishing features between these two types of cosmic string networks. Some suggestions in this regard have been put forward in the literature [18 − 20].

Here we explore the possibility that these strings were able to propagate in the extra dimensional part of the geometry. It is clear that this could have important consequences for the evolution of the network of strings [20]. In this paper we demonstrate this fact by finding stable solutions for 4-dimensional extended closed loops that wind around a circle of the internal space. We show that even though the extra dimensional part of the internal geometry in this region is a three sphere threaded with fluxes, the solutions remain stable for some range of the parameters. On the other hand, these light degrees of freedom that parametrize the position of the string on the compact space can also be thought of as a neutral current flowing along the string. This current has a very peculiar equation of state that could in principle help us to distinguish between fundamental versus field theory cosmic strings.

The organization of the paper is the following. In section 2 we present the simple solutions of strings propagating in $M^4 \times S^1$, and discuss its connection to the superconducting string case. Section 3 discusses the embedding and stability of these solutions in the deformed conifold geometry. Finally in the conclusions we elaborate on the possible role that these solutions may play within the recently proposed scenarios of brane inflation.

2. Strings on $M^4 \times S^1$

The Nambu-Goto action of a string propagating in D+1 dimensions is given by,

$$S = -\mathcal{T} \int \sqrt{-\gamma}\; d\sigma dt\,, \tag{2.1}$$

where $\mathcal{T} = 1/2\pi\alpha'$ is the tension of the string, $\gamma_{ab} = g_{MN}\partial_a X^M \partial_b X^N$ is the induced metric on the worldsheet parametrized by the intrinsic coordinates, σ and t, and $X^M(\sigma, t)$ gives the embedding of the string motion in the D+1-dimensional space-time. The equations of motion in the conformal gauge for a string propagating in a spacetime with metric g_{MN} are [16],

$$\ddot{X}^M - X''^M + \Gamma^M_{NP}(\dot{X}^N \dot{X}^P - X'^N X'^P) = 0 \tag{2.2}$$

where X' and $\dot{X}$ denote the derivatives with respect to σ and t and Γ^M_{NP} are the Christoffel symbols for the background metric. On the other hand, in order to be in the conformal gauge, we have to impose on the solutions the following constraints:

$$\dot{X}^M X'_M = 0\,, \tag{2.3}$$
$$\dot{X}^M \dot{X}_M + X'^M X'_M = 0\,. \tag{2.4}$$

We can now particularize these equations to the case at hand, a 4+1 dimensional spacetime with one of the spatial dimensions compactified on a circle. In this case, the metric is:

$$ds^2 = g_{MN} dX^M dX^N = -dt^2 + dx_i dx^i + d\theta^2\,. \tag{2.5}$$

In this background, we can use the extra gauge freedom to choose $X^0 = t$ so that the 4-dimensional spatial vector X^J that parametrizes the position of the string satisfies the following set of equations:

$$\ddot{X}^J = X^{J''}\,, \tag{2.6}$$
$$\dot{X}^J X'_J = 0\,, \tag{2.7}$$
$$\dot{X}^J \dot{X}_J + X'^J X'_J = 1\,. \tag{2.8}$$

We look for solutions of these equations that describe macroscopic strings extended along the non compact 3+1 dimensional part of this spacetime and winding the transverse circle, namely solutions of the following form,

$$x^0(\sigma, t) = t\,, \tag{2.9}$$
$$\mathbf{x}(\sigma, t) = \mathbf{a}(\sigma - t)\,, \tag{2.10}$$
$$\theta(\sigma, t) = \sigma + (\sigma - t) b\,, \tag{2.11}$$

where $\mathbf{a}$ is an arbitrary vector function and b a constant. It is clear that this ansatz fulfills the equations of motion, so we only have to impose the constraint equations which are also fulfilled provided that,

$$b(b+1) + |\mathbf{a}'|^2 = 0\,. \tag{2.12}$$

Note that we also have to choose b such that it respects the periodicity of θ.

This solution describes a wiggle of arbitrary shape propagating on a straight string along the extra dimension. Solutions of this type have been known in the cosmic string literature for quite some time [21] and generalize the solutions found in the circular case in [22]. They have also been discussed recently within string theory in [23]. From the 4-dimensional perspective, this string looks like a loop of arbitrary form stabilized by the backreaction on its worldsheet of the perturbations of the string along the extra dimension. In this regard, they are basically a compactified version of the wiggly cosmic string model [24, 25].

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We can also think of these wiggles along the compactified directions as an induced current on the worldsheet [26], and therefore consider these strings as a model of neutral superconducting strings [27]. Hence, it is not so surprising that there are stable configurations of string loops since states like this are well known in the context of superconducting cosmic string models [28]. Also the arbitrary shape for the strings in this case is easily understood due to the fact that the 4-dimensional string tension vanishes in those configurations [29, 30].

3. Strings on the deformed conifold

We now consider solutions of the type described in the previous section propagating in the background of the warped deformed conifold of type IIB supergravity [9]. In this solution the dilaton field is constant, $\mathrm{e}^{\Phi} = g_s$, and the line element, 3-form and self-dual 5-form Ramond-Ramond field strengths, and the Kalb-Ramond 2-form are given by

$$\begin{aligned} ds_{10}^2 &= h^{-1/2}(\tau) dx^\mu dx_\mu + \\ &+ \frac{1}{2} h^{1/2}(\tau) \epsilon^{4/3} K(\tau) \left[\frac{1}{3K^3(\tau)} \left(d\tau^2 + g_5^2 \right) + \right. \\ &\left. + \cosh^2\left(\frac{\tau}{2}\right) \left(g_3^2 + g_4^2 \right) + \sinh^2\left(\frac{\tau}{2}\right) \left(g_1^2 + g_2^2 \right) \right] , \qquad (3.1) \\ F_3 &= \frac{1}{2} M\alpha' \left\{ g_5 \wedge g_3 \wedge g_4 + d\left[\frac{\sinh\tau - \tau}{2\sinh\tau} \left(g_1 \wedge g_3 + g_2 \wedge g_4 \right) \right] \right\} , \qquad (3.2) \\ \tilde{F}_5 &= \mathcal{F}_5 + \star\mathcal{F}_5 , \qquad \mathcal{F}_5 = B_2 \wedge F_3 , \qquad (3.3) \\ B_2 &= g_s M\alpha' \frac{\tau\coth\tau - 1}{4\sinh\tau} d\tau \wedge \left((\cosh\tau - 1) g_1 \wedge g_2 + (\cosh\tau + 1) g_3 \wedge g_4 \right) , \qquad (3.4) \end{aligned}$$

where

$$\begin{aligned} K(\tau) &= \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3}\sinh\tau} , \qquad (3.5) \\ h(\tau) &= (g_s M\alpha')^2 2^{2/3} \epsilon^{-8/3} \int_\tau^\infty dx \, \frac{x\coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3} , \qquad (3.6) \end{aligned}$$

and the 1-forms $g_1, \ldots g_5$ are linear combinations of the Cartan 1-forms on the coset $\mathrm{SU}(2)\times \mathrm{SU}(2)/\,\mathrm{U}(1)$ [31], namely:

$$\begin{aligned} g_1 &= \frac{1}{\sqrt{2}}(e_1 - e_3) , \qquad g_2 = \frac{1}{\sqrt{2}}(e_2 - e_4) , \qquad (3.7) \\ g_3 &= \frac{1}{\sqrt{2}}(e_1 + e_3) , \qquad g_4 = \frac{1}{\sqrt{2}}(e_2 + e_4) , \qquad g_5 = e_5 , \qquad (3.8) \\ e_1 &= -\sin\theta_1 d\phi_1 , \qquad e_2 = d\theta_1 , \qquad (3.9) \\ e_3 &= \cos\psi \sin\theta_2 d\phi_2 - \sin\psi d\theta_2 , \qquad (3.10) \\ e_4 &= \sin\psi \sin\theta_2 d\phi_2 + \cos\psi d\theta_2 , \qquad (3.11) \\ e_5 &= d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 \ . \qquad (3.12) \end{aligned}$$

In this parametrization, $0 \le \theta_i < \pi$, $0 \le \phi_i < 2\pi$, $0 \le \psi < 4\pi$ for $i = 1, 2$.

We are interested in the dynamics of strings near the bottom of the conifold, thus we consider the $\tau \to 0$ limit of the supergravity solution. The metric becomes (choosing the deformation parameter $\epsilon = 12^{1/4}$)

$$
\begin{aligned}
ds^2_{10} = k^{-1}\left(1+\frac{\tau^2}{a_0 6^{4/3}}\right) dx^\mu dx_\mu + \\
+k\Bigg[\frac{1}{2}\left(1+\left(\frac{1}{5}-\frac{1}{a_0 6^{4/3}}\right)\tau^2\right)\left(d\tau^2+g_5^2\right)+ \\
+\left(1+\left(\frac{3}{20}-\frac{1}{a_0 6^{4/3}}\right)\tau^2\right)\left(g_3^2+g_4^2\right)+\frac{\tau^2}{4}\left(g_1^2+g_2^2\right)\Bigg]+\mathcal{O}(\tau^3)\,,
\end{aligned}
\tag{3.13}
$$

where $k = a_0^{1/2} 6^{-1/3} g_s M \alpha'$ and $a_0 \approx 0.718$.

At $\tau = 0$ the angular part of the conifold degenerates to a round three- sphere of radius $k^{1/2}$,

$$
d\Omega_3^2 = \frac{1}{2} g_5^2 + g_3^2 + g_4^2\,,
\tag{3.14}
$$

plus a collapsed two-sphere,

$$
d\Omega_2^2 = g_1^2 + g_2^2\,.
\tag{3.15}
$$

The stability group of the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetric solution of the conifold defining equations is enhanced from U(1) to a full SU(2) at $\tau = 0$ [8] which we use to set $\theta_2 = \phi_2 = 0$. Therefore, it is easy to see that (3.14) is indeed the metric on a round three-sphere as $d\Omega_3^2 = Tr\ dT^\dagger dT/2$ for the SU(2) matrix

$$
T = \begin{pmatrix} \cos\frac{\theta_1}{2} e^{i\frac{\psi+\phi_1}{2}} & \sin\frac{\theta_1}{2} e^{i\frac{-\psi+\phi_1}{2}} \\ -\sin\frac{\theta_1}{2} e^{i\frac{\psi-\phi_1}{2}} & \cos\frac{\theta_1}{2} e^{-i\frac{\psi+\phi_1}{2}} \end{pmatrix}
\tag{3.16}
$$

and (3.15) is just $d\theta_1^2 + \sin^2\theta_1 d\phi_1^2$.

We want to study strings propagating in this background which form closed loops in $I\!R^{3,1}$ and wind around a maximal circle on the blown-up three-sphere. We parametrize the string by target space coordinates $(x^0, \ldots, x^3, \tau, \theta_1, \psi, \phi_1, \theta_2, \phi_2)$ and we use the worldsheet gauge freedom to fix $X^0 = t$ and $(X^1/X^2) = \tan\sigma$. Within this gauge we take the following ansatz:

$$
X^M = (t, r(t)\,\sin\sigma, r(t)\,\cos\sigma, z_0, \tau, 0, 2n_\psi\sigma + \varphi(t), n_\phi\sigma + \varphi(t), 0, 0)\,,
\tag{3.17}
$$

where n_ψ and n_ϕ are integers labeling the winding in the ψ and ϕ directions respectively (for $0 \le \sigma < 2\pi$, $2n_\psi\sigma$ runs n_ψ times over the range of ψ). Let us mention that circular string solutions winding around transverse spheres are known [32, 33] albeit not for the case of the warped background considered here.

In what follows we will focus on solutions with fixed radius r and find the conditions for their stability for the case of both a fundamental string and a D-string.

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3.1 F-string

We consider the Nambu-Goto action for the bosonic sector of the F-string:

$$S = -\mathcal{T} \int d\sigma dt \, \sqrt{-\gamma} + \mu \int B_2 \,, \tag{3.18}$$

where the charge $\mu = \mathcal{T}$ and B_2 stands for the Kalb-Ramond two form. Two conserved quantities follow from the energy momentum tensor namely,[1]

$$E = 2\pi\alpha' k \left(\frac{\delta \mathcal{L}}{\delta \dot{X}^I} \dot{X}^I - \mathcal{L} \right) = \frac{r^2 + \frac{1}{2}k^2 s'^2}{\sqrt{(1-\dot{r}^2)\left(r^2 + \frac{1}{2}k^2 s'^2\right) - \frac{1}{2}k^2 r^2 \dot{s}^2}} \,, \tag{3.19}$$

$$\ell = 2\pi\alpha' k \frac{\delta \mathcal{L}}{\delta \dot{X}^I} (X^I)' = \frac{\frac{1}{2}k^2 s' \dot{s} r^2}{\sqrt{(1-\dot{r}^2)\left(r^2 + \frac{1}{2}k^2 s'^2\right) - \frac{1}{2}k^2 r^2 \dot{s}^2}} \,, \tag{3.20}$$

where the index I labels the eight coordinates not fixed by the gauge choice and we have defined for convenience[2] $s \equiv \psi + \phi_1$. The r.h.s. of (3.19) and (3.20) are obtained by specializing to the ansatz (3.17). These two conserved quantities imply that the circular orbits have radius $r = \sqrt{\ell}$ and transverse angular velocity $\dot{\varphi} = 1/\sqrt{2}k$ for any non vanishing winding numbers n_ψ, n_ϕ. It is clear that these solutions are stable with respect to 4-dimensional radial perturbations due to the fact that they are basically the static solution of a 1-dimensional analogous problem on the minimum of its potential.

Unlike the case of the previous section, in which the stability of the winding in the transverse direction is topological, the solutions obtained above are classically stable due to the dynamics in the transverse directions. They turn out to be stable if the radius r of the string is large enough. From the expansion (3.13) it is not difficult to see that the solution is stable under a small deviation in the τ direction, $\delta\tau(t)$, from the bottom of the conifold ($\tau = 0$). $\delta\tau$ contributes positive definite kinetic and potential energies to the action. Turning on a small perturbation $\delta\theta$ in the coordinate θ, introduces a perturbation in the action (to second order in $\delta\theta$) proportional to

$$\left(\ell + \frac{1}{2}k^2 (2n_\psi + n_\phi)^2 \right) \dot{\delta\theta}^2 - \frac{1}{2k^2} \left(\ell - \frac{1}{2}k^2 (2n_\psi + n_\phi)^2 \right) \delta\theta^2 \,, \tag{3.21}$$

with no mixing with the τ perturbations (whose lowest order is quadratic). We also notice that there is no contribution to this order from perturbations in the other transverse directions.[3] Therefore, the solution is stable as long as

$$\ell > \frac{k^2 (2n_\psi + n_\phi)^2}{2} \,. \tag{3.22}$$

[1] Note that B_2 vanishes at the bottom of the conifold.

[2] The fact that $s/2$ plays the role of the angular coordinate on the S^1 case, and therefore that the solution should be periodic in this variable, implies that only even n_ϕ should be considered.

[3] The decoupling of the fluctuations in the τ, ϕ_2, θ_2 from the directions on the 3−sphere to quadratic order is, *a priori*, nontrivial. Therefore, the stability analysis here performed could not have been inferred from the one of [32, 33] even though we arrive at similar results.

Before we end this subsection, let us mention that a σ dependence on the fluctuations considered does not change the result. Indeed, by considering

$$\delta\theta(\sigma,t) = \sum_n \mathrm{e}^{in\sigma} f_n(t)\,, \qquad f_n = f_{-n}\,, \tag{3.23}$$

leads to an increase in the potential energy contribution of the n-th mode proportional to n^2, therefore improving the stability.

3.2 D-string

The action for a D-string is the sum of the Born-Infeld and Chern-Simons terms which, in the absence of a world-sheet gauge field, reads

$$S = -\mathcal{T}\int d\sigma dt\ \mathrm{e}^{-\Phi}\sqrt{-\det[\gamma + B_2]} + \mu\int C_2\,, \tag{3.24}$$

where the charge is related to the tension as $|\mu| = \mathcal{T}$ and C_2 stands for the Ramond-Ramond two form. In the ansatz (3.17) for $\tau = 0$ the equations of motion coincide with those of the F-string since the contribution from the F_3 form vanishes in this case. However, the Chern-Simons term alters the stability condition under small θ fluctuations. It becomes:

$$\ell > \frac{1}{2}k^2\left[(2n_\psi + n_\phi)^2 \pm (2n_\psi^2 - n_\phi^2)\right]\,, \tag{3.25}$$

where $\pm = \mu/|\mu|$; while the stability under τ fluctuations is guaranteed for any choice of parameters as in the F-string case.

3.3 Generalizations

We expect the ansatz (3.17) to belong to wider class of stable classical string solutions propagating at the bottom of the conifold geometry. In particular, we can also find solutions for the equations of motion for F and D strings with arbitrary four dimensional shape if we wind the string along a maximal circle of the S^3. In order to see that, it is sufficient to look at the general equations of motion in the conformal gauge for a string in the $\tau \to 0$ limit of our geometry. Also it is more convenient to use the familiar parametrization of the sphere, namely

$$ds^2 = k^{-1}dx^\mu dx_\mu + k\left(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)\right)\,. \tag{3.26}$$

If we now restrict ourselves to an ansatz of the form, $(t, x^i, 0, \pi/2, \pi/2, \phi, 0, 0)$ then we see that the equations of motion (2.2) for the F and D string in this background are identical to the solutions found for the $M^4 \times S^1$ case and therefore allow for any shape for the string in four dimensions. The stability of these solutions with respect to perturbations along the sphere follows from the arguments on previous sections.

4. Concluding remarks

We have found stable solutions for loops of F and D strings extended along the uncompactified four dimensional space and winding a circle of the transverse 3-sphere at the bottom of the warped deformed conifold. The reason for its stability from the four dimensional point of view is the modification of the string dynamics due to the massless degrees of freedom present on the string associated with the excitations along the transverse directions. This is reminiscent of what happens in the case of superconducting cosmic string case, where solutions of this type are known to exist, the so called vorton solutions [28]. In fact, we can regard the strings propagating at the tip of this geometry as neutral superconducting strings, as long as the extra dimensional manifold there remains a 3-sphere. If this was the case on a realistic model of brane inflation, we should carefully study the cosmological implications of our findings. The existence of these stable loops that would not decay by gravitational radiation, would impose important constraints on the string network evolution [34].

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